Suppose $f(x, y)$ is a nice, differentiable function that we are trying to optimize subject to the constraint $x^2 + y^2 - 1 = 0$. If we try to solve the Lagrange multipliers equations, which of the following statements is accurate?

- It is possible to find no solutions.
- We are guaranteed to find at least one solution.
- We are guaranteed to find at least two solutions.
- None of the above

Total Results: 13

Example: $f(x,y) = x+y$ $\qquad g(x,y) = x^2+y^2-1$

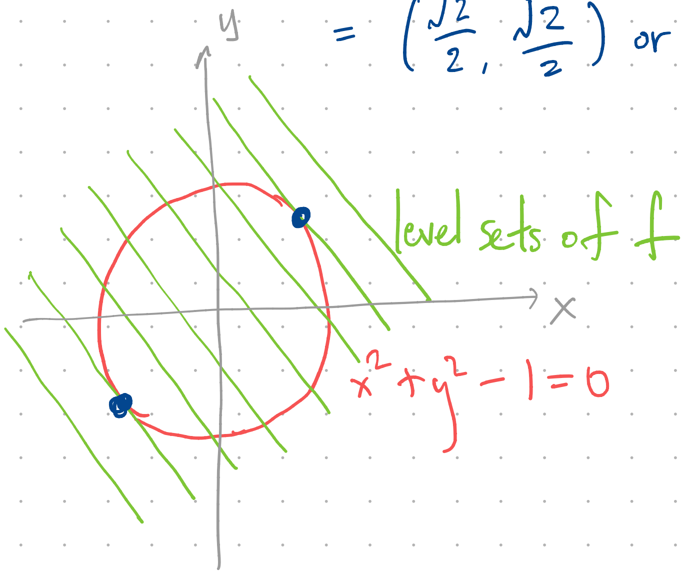
$\nabla f = \langle 1, 1 \rangle \qquad \nabla g = \langle 2x, 2y \rangle$

$$\begin{cases} 1 = 2x\lambda \\ 1 = 2y\lambda \\ x^2+y^2-1=0 \end{cases}$$

Get $x=y$ from first two eqs...

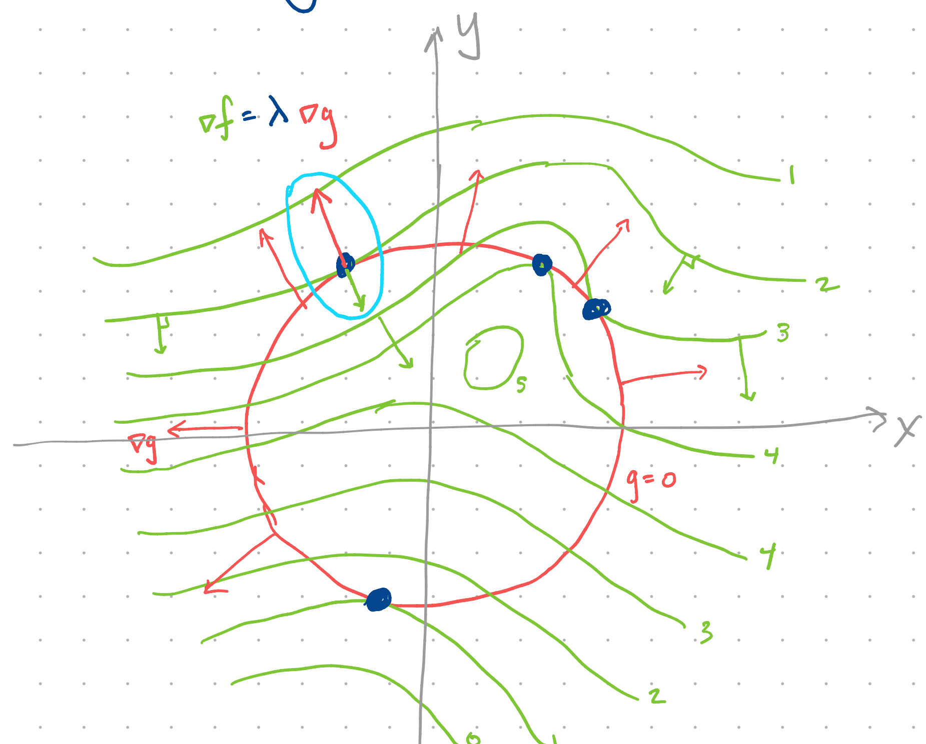
Then solve to find (x,y)

$$= \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \text{ or } \left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$$



For this particular choice of $f(x,y)$, there were <u>two</u> solutions.

What can you say in general if $f(x,y)$ is allowed to be any (differentiable) function?

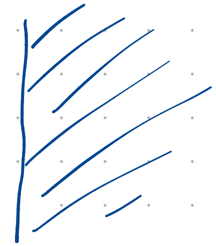
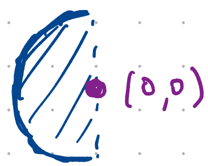
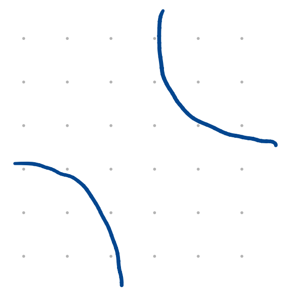


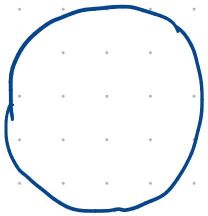
(Another example, this time with 4 solutions)

Key point: EXTREME VALUE THM.

it guarantees the existence of max and min of a function f on a region R if....

- f is continuous on R, and
- R is closed: every point that is "infinitesimally close" to R is actually in R. In other words, any point not in R has "positive distance" from R.
- R is bounded, i.e. doesn't go "off to infinity".

regions	closed	bounded
 $x \geq 0$	✓	✗
 $x^2+y^2 \leq 1$ & $x<0$	✗	✓
 $xy=1$	✓	✗

	closed	bounded
	✓	✓

$x^2+y^2-1=0$

Moreover: $f(x,y)$ was assumed differentiable, so in particular continuous

So <u>EVT applies</u>.

$\therefore$ $f(x,y)$ has a max and a min on ○.

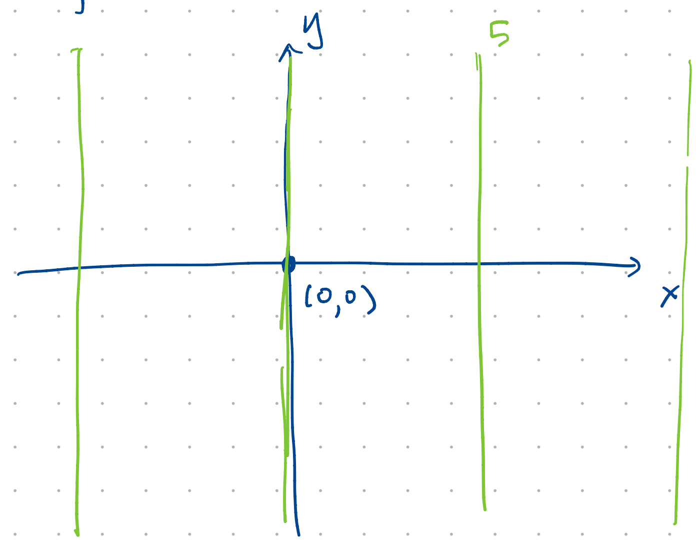
Note: if max and min occur @ same point, then f is constant on constraint region, so <u>every pt</u> is a solution (∞ many sol.)

In any case: at least <u>two</u> solutions.

⚠ If $g(x,y)=0$ is not closed or not bdd, this is not applicable. e.g. $f(x,y)=x+y$ has no max and no min on $x-y=0$

Every point (in the domain of f) by definition belongs to some level set of f. I just can't draw <u>all</u> the level sets (there are infinitely many).

$f(x,y)=4+\cos x$



$f(0,0)=5$ so $(0,0)$ is on level set $4+\cos x=5$